TCU Math Newsletter

Anything that, in happening, causes itself to happen again, happens again.

- Douglas Adams

NSF Research Experience for Undergraduates Summer 2023 Programs

The National Science Foundation (NSF) funds summer research opportunities for mathematics undergraduate students through REU Sites across the country. Students are granted stipends and, in most cases, housing and a travel allowance. The application deadlines vary, but some are in February 2023.

A list of Mathematics REU sites where you can find details and learn about the individual programs and the application processes can be found at www.nsf.gov/crssprgm/reu/list_result.jsp?unitid=5044

Another REU program that mathematics students may be interested in is the NSF REU in Data Science at Worcester Polytechnic Institute. Information about this REU can be found at

www.wpi.edu/academics/departments/data-science/reu-program

Texas Oklahoma Regional Undergraduate Symposium on February 18

The 15th Annual Texas Oklahoma Regional Undergraduate Symposium (TORUS) will be held at Dallas College Brookhaven Campus on Saturday, February 18, 2023. Undergraduate students, faculty and anyone interested in the mathematical sciences are encouraged to attend. The symposium features research presentations by undergraduates, networking with others in the field, information about jobs in mathematics, and a Jeopardy-style contest. Lunch will be provided for all participants. Attendees who plan to present must register by February 10, but others can register later or at the conference. More information and a link to register can be found at

https://www.dallascollege.edu/cd/schools/stem/events/pages/torus-conference.aspx?utm_source=shortcut-url&utm_medium=redirect&utm_campaign=shortcuts&utm_content=torus

TCU Career & Intern Expo

TCU's all majors Career & Intern Expo Spring 2023 will be held on Wednesday, February 22, 2023 from 3:00 pm to 6:00 pm in the TCU Recreation Center.

Over 100 employers will be participating and recruiting for summer internships and full-time employment.

Students can register for the expo at
https://tcu.joinhandshake.com/career_fairs/36104/student_preview?token=5HeXFeRekesKio0CBMN4IUBp2H_WHYACnGBvcDU-Ivyoza3U-LQ4eA
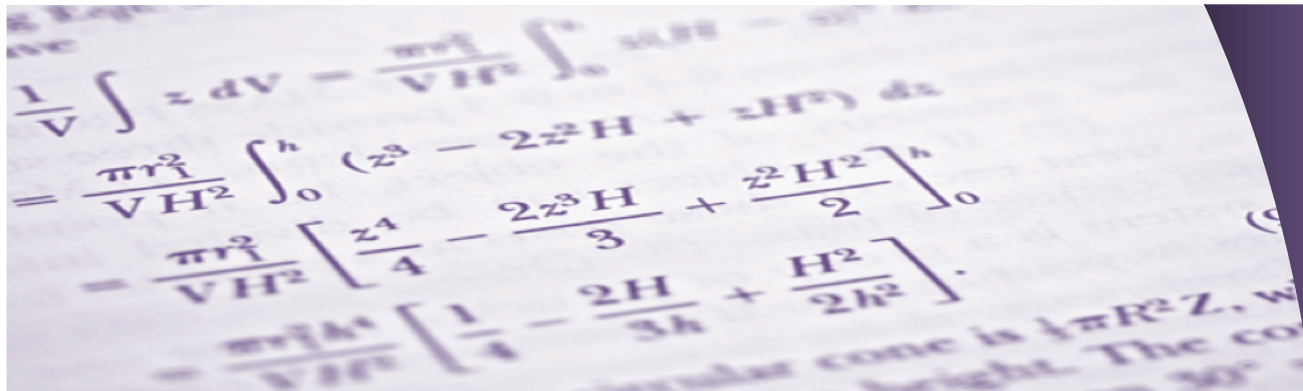
Solution to the November 2022 Problem of the Month

Problem: What is the largest possible area of a quadrilateral with sides of lengths 1, 1, 1, 2?

Solution: The largest possible area is $3\sqrt{3}/4$.

Given a concave quadrilateral with sides of lengths 1, 1, 1, 2, we could obtain one with larger area by reflecting across the diagonal that is outside the quadrilateral. Thus, we need only consider convex quadrilaterals, which we label as in the figure.
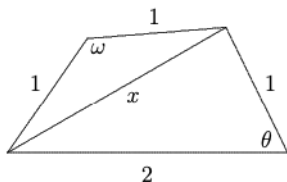
By the law of cosines, $x^2 = 5 - 4\cos\theta = 2 - 2\cos\omega$. Because $1 \le x \le 2$, we have $0 \le \theta \le \arccos(1/4)$. The area is $\sin\theta + \frac{1}{2}\sin\omega$. Rather than eliminating one variable, which introduces square roots, we use Lagrange multipliers,

$$A = \sin\theta + \frac{1}{2}\sin\omega - \lambda(4\cos\theta - 2\cos\omega - 3).$$

Differentiating,

$$A_\theta = \cos\theta + 4\lambda\sin\theta = 0,$$
$$A_\omega = \frac{1}{2}\cos\omega - 2\lambda\sin\omega = 0,$$
$$A_\lambda = -(4\cos\theta - 2\cos\omega - 3) = 0.$$

From the first two equations, we find $\tan\theta = -1/(4\lambda)$ and

$$\tan\omega = 1/(4\lambda) = -\tan\theta.$$

We conclude $\omega = \pi - \theta$. The third equation now implies $\cos\theta = 1/2$ or $\theta = \pi/3$, with area $3\sqrt{3}/4$. The areas at the endpoints are $\sqrt{3}/4$ and $\sqrt{15}/4$, so our critical point yields the maximal area.

This month's problems was solved by Duc Toan Nguyen.

February 2023 Problem of the Month

A list of n whole numbers, none larger than 2023 and whose sum is at least $2022n + 1$, is arranged from largest to smallest. Show that the kth number on the list is at least $2024 - k$.