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TCU Math Newsletter

I suppose I'm a theory builder or maybe a conjecture maker. I'm not a conjecture prover very much, but I don't know. It's true that I'm not good at solving problems. For example, I would never be good in the Math Olympiad. There speed counts and I am certainly not a speedy worker. That's one pleasant thing in mathematics: It doesn't matter how long it takes if the end result is a good theorem. Speed is an advantage, but it is not essential.

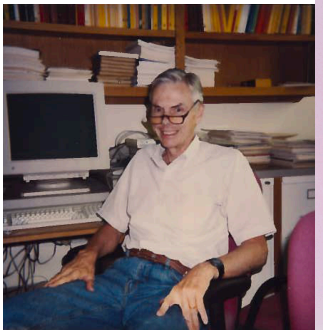
- John T. Tate

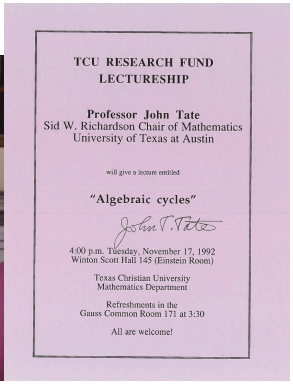
John Tate (1925-2019)

Prominent American mathematician John Tate died on October 16, 2019 at the age of 94. He made fundamental contributions to algebraic number theory and arithmetic geometry and won the prestigious Abel Prize in 2010.

Professor Tate was at Harvard for 36 years. He took the position of Sid W. Richardson Foundation Regents Chair at the University of Texas at Austin in 1990 where he stayed until his retirement in 2009.

TCU was honored to have Professor Tate speak in the TCU Math Colloquium series in 1992. The program for his appearance and a picture taken of him at TCU on that day are shown here.





TCU RESEARCH FUND
LECTURESHIP

Professor John Tate
Sid W. Richardson Chair of Mathematics
University of Texas at Austin

will give a lecture entitled

"Algebraic cycles"

4:00 p.m. Tuesday, November 17, 1992
Winton Scott Hall 145 (Einstein Room)

Texas Christian University
Mathematics Department

Refreshments in the
Gauss Common Room 171 at 3:30

All are welcome!

Colloquium Talk on November 5

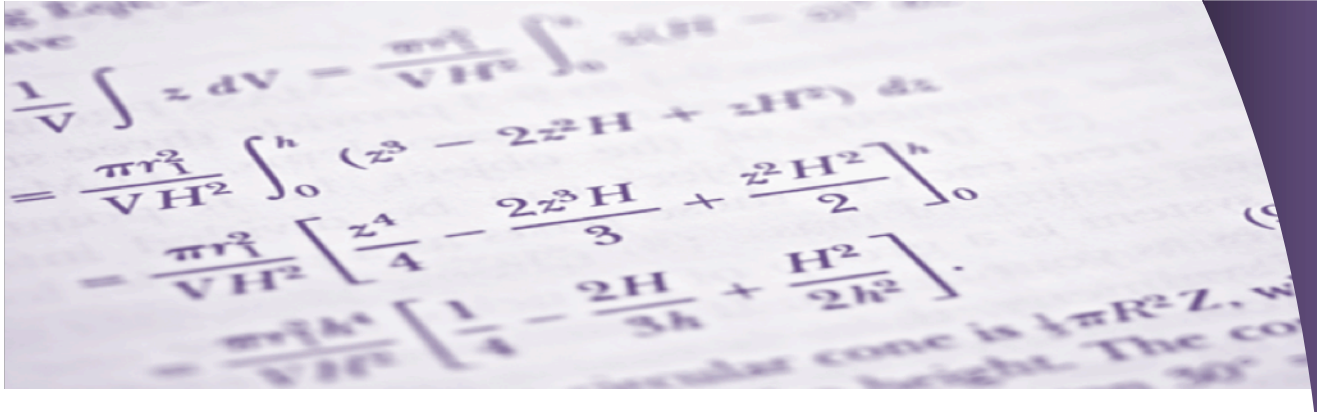
Dr. Sarah Frei of Rice University will present the talk "Moduli spaces of sheaves on K3 surfaces" in the TCU Math Colloquium series on Tuesday, November 5. Her talk is at 3:30 pm in TUC 243, and refreshments will be served in TUC 300 at 3:00 pm.

Gamma Iota Sigma Chapter Established at TCU

The Gamma Xi chapter of the national actuarial group Gamma Iota Sigma was established at TCU in September 2019. The first initiation ceremony for new members of the chapter was held on October 9. The student initiates were: Alaina Jerguson (President), Erin Cahalan (Business and Risk Management Chair), Maddie Graff (Secretary), Abarrane Henderson- (treasurer), Kiley Tomes (Industry Relations Chair), Cade Strephans, Anna Breck, Dechen Yonten, Laura Hubbard, Lillian Langevin, Abby Searles, Michelle Hearn, Nicholas Isaacks, Jacie Mascarenhas, Carneisha Daniels, Sarah Darley , Nichole Watkins, Amanda Norman, Emma Schmidt, Joshua Randle El, Kathleen Ravalico, Sam Kennemer, Franciso Avelar, Emma Medlin, Hien Cao, Minh Bui, Kai Qin, Hung Tran, Dashiva Francois, Eric Haacker, Kieutrinh Ha, Blake Herbst, and Casey Lutz. A picture taken at the initiation ceremony is shown below.

TCU Math Club Meeting

The TCU Math Club will meet at 5 pm on Tuesday, November 19 in TUC 300. All TCU students interested in mathematics are encouraged to come.



Solution to the October 2019 Problem of the Month

Problem: Are there integers x and y (possibly negative) and $n \geq 2$ such that $x^n + y^n = 2019$?

Solution: No. Because $x^{pq} = (x^q)^p$, it suffices to prove there are no solutions when n is prime. The prime factorization of 2019 is $3 \cdot 673$. Write x and y in the form $3k + r$, where r is one of $-1,0,1$. Then $(3k + r)^2 = 3(3k^2 + 2kr) + r^2$. In order for $x^2 + y^2$ to be divisible by 3, both x and y must be divisible by 3, hence $x^2 + y^2$ must be divisible by 9, a contradiction. Similarly, $(3k + r)^3 = 9(3k^3 + 3k^2r + kr^2) + r$, so $x^3 + y^3$ is divisible by 9 if it is divisible by 3.

We may assume $x \geq y$ and, since we are left with $n \geq 5$ and odd, rewrite the equation in the more convenient form $x^n - z^n = 2019$. The only possible powers of 2, 3, 4, … between $2019/2$ and $2019 + 1^n$ is $4^5 = 1024$. Because 2019-1024 is not a perfect power, we conclude $z \geq 2$. If $x - z \geq 3$, then $x^n - z^n \geq 5^5 - 2^5 = 3093 > 2019$.

Therefore, $x - z = 1$. However, in that case

$$x^n - z^n \equiv x - z = 1 \pmod 3,$$

our final contradiction.

This month's problem was solved by Brad Beadle ('96) and Klaus Kienzle.

November 2019 Problem of the Month

Let $a_0 \leq b_0 \leq c_0$ be real numbers. From $a_n \leq b_n \leq c_n$, replace either a_n or c_n with the average of the three numbers and sort them obtaining three numbers satisfying $a_{n+1} \leq b_{n+1} \leq c_{n+1}$. Will the sequences $(a_n), (b_n), (c_n)$ always converge?

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

Editor: Rhonda Hatcher
Problem Editor: George Gilbert
Thought of the Month Editor: Robert Doran