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# TCU Math Newsletter

Whoever despises the high wisdom of mathematics nourishes himself on delusion.

- Leonardo da Víncí

#### **TCU Calculus Bee Winners**

The annual TCU Mathematics Department Calculus Bee was held on Monday, April 22, 2019. The first-place winner was Cong Minh Quang Truong, second place went to Bao Thach, and Lauren Nagel was the third-place winner. Congratulations to these students!



From left to right: Lauren Nagel, Bao Thach, and Cong Minh Quang Truong.

## **Colloquium Talk on April 26**

Dr. Keaton Hamm of the University of Arizona will be the first speaker this semester in the TCU Colloquium Series on Friday, September 20 at 3:30 pm in TUC 243. Refreshments will be served in TUC 300 at 3:00 pm.

#### **New Mathematics Department Chair**

After 8 years of service, Dr. George Gilbert transitioned out of the position of TCU Mathematics Department Chair. Starting with the Fall 2019 semester, the new Chair is Dr. Greg Friedman.

#### **Putnam Mathematics Contest**

The 80th Annual William Lowell Putnam Mathematical Competition will be held on Saturday, December 7, 2019, from 9:00 am to noon and 2:00 to 5:00 pm. The questions require different levels of mathematical background, and all require a bit of ingenuity to solve. The scores on the exam are typically quite low, and even answering a couple of questions is considered an excellent performance. The competition is open to undergraduates enrolled in colleges and universities of the United States and Canada who have not yet received a college degree. For more information about the contest visit http://math.scu.edu/putnam/.

Professor George Gilbert will be holding several practice sessions for interested students. The sessions will be independent, so you can attend as many or as few your schedule allows. Students interested in either the practice sessions or in signing up to take the Putnam exam this year should contact Dr. Gilbert at <u>g.gilbert@tcu.</u>

### Actuarial Career Event at TCU on September 25

On Wednesday, September 25, TCU will host an Actuarial Career Event in TUC 139 from 12:00 pm to 2:00 pm. All students interested in this career option should attend this event. For more information, contact Dr. Susan Staples at <u>s.staples@tcu.edu</u>.

#### Math Résumé Book Deadline

Wednesday, September 18 is the deadline for résumé submission for the 2019 Math Résumé Book. Students must make an appointment with Gabriela Pineider to have their résumé reviewed in order to be included in the book. Her email address is gabriela.pineider@tcu.edu.

# Solution to the April 2019 Problem of the Month

**Problem:** Let *n* be a positive integer. Determine the number of real roots of  $1 + 2x + 3x^2 + \dots + (n-1)x^{n-2} + nx^{n-1}$ 

as a function of *n*.

**Solution:** There are no real roots when *n* is odd and one real (negative) root when *n* is even. By the formula for the sum of a geometric progression, we have

$$1 + 2x + 3x^{2} + \dots + (n-1)x^{n-2} + nx^{n-1} = \frac{d}{dx} (1 + x + x^{2} + \dots + x^{n-1} + x^{n})$$
$$= \frac{d}{dx} \frac{x^{n+1} - 1}{x - 1} = \frac{nx^{n+1} - (n+1)x^{n} + 1}{(x - 1)^{2}}.$$

Noting that any real roots of the original polynomial must be negative, we see that the roots of the polynomial are the negative roots of  $nx^{n+1} - (n+1)x^n + 1$ . If it had at least two negative roots, the mean value theorem would imply that

$$\frac{d}{dx}\left(nx^{n+1} - (n+1)x^n + 1\right) = (n^2 + n)(x-1)x^{n-1}$$

has a negative root between them, which is doesn't. Thus the polynomial has at most one root. (One can draw the same conclusion from Descartes' rule of signs.) Looking at the value of the polynomial at 0 and its limits as x approaches negative infinity, we see that the polynomial has an even number of negative roots for n odd and an odd number of negative roots for n even.

This month's problem was solved by Brad Beadle ('96).

# **September 2019 Problem of the Month**

A deck has n red cards, n white cards, and n blue cards. Three cards are drawn at random. The probability that all three are different is an integer k times the probability all three are the same. Find n and k.

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

Editor: Rhonda Hatcher Problem Editor: George Gilbert Thought of the Month Editor: Robert Doran