



TCU Math Newsletter

In the broad light of day mathematicians check their equations and their proofs, leaving no stone unturned in their search for rigour. But, at night, under the full moon, they dream, they float among the stars and wonder at the miracle of the heavens. They are inspired. Without dreams there is no art, no mathematics, no life.

- Michael Atiyah (Fields Medalist)

TCU Math Club Meeting on Wednesday, March 21

Dr. Susan Staples will be presenting a talk for The TCU Math Club from 3:30 to 4:30 pm on Wednesday, March 21 in TUC 245. Refreshments will be served before the talk in TUC 300 starting at 3:00 pm.

The title of Dr. Staples' talk is "Series Shenanigans." Her description of the talk is:

Starting from Calculus notions on sequences and series, we merrily prove a number of wonderful results. Unfortunately, some of our results will be false. We hope by the end of the talk to figure out what went wrong and, in turn, learn how some of these mistakes led to exciting and useful mathematics!

All TCU students, faculty, and other interested members of the community are welcome to attend.

TCU Colloquium Talk on March 27

The next talk in Frank Stones Colloquium Series will be presented by Professor Brian Rainies of Baylor University. His talk will be on Tuesday, March 27 at 3:30-4:30 pm in TUC 243. Students, faculty, and members of the community are invited to attend the talk and enjoy refreshments served in TUC 300 during the half hour before the talk.

Calculus Bee on Monday, April 9

The annual TCU Mathematics Department Calculus Bee will be held on Monday, April 9 at 3:30 pm in TUC 245. The material covered is Calculus I and II, but not beyond the material that current Calculus II students have had. Those wishing to compete should arrive for refreshments and contestant number assignment between 3:00 and 3:20 pm in TUC 300. (Those with a class conflict can simply go straight to TUC 245 at 3:30pm.) All TCU undergraduates are eligible to compete. TCU bookstore gift cards will be awarded to the top three finishers, with \$75 for first place, \$50 for second place, and \$25 for third place.



Solution to the February Problem of the Month

Problem: Let n be an integer greater than 1 and let x_1, x_2, \dots, x_n be positive numbers. Prove that

$$\sum_{1 \leq i < j \leq n} \frac{1}{x_i + x_j} \leq \frac{n-1}{4} \sum_{1 \leq i \leq n} \frac{1}{x_i}.$$

Solution: First consider the case $n = 2$. Clearing denominators, the inequality

$$\frac{1}{x_1 + x_2} \leq \frac{1}{4} \left(\frac{1}{x_1} + \frac{1}{x_2} \right)$$

is equivalent to $4x_1x_2 \leq (x_1 + x_2)^2$, which is equivalent to $0 \leq (x_1 - x_2)^2$, which holds.

We apply the case $n = 2$ to the general case, obtaining

$$\sum_{1 \leq i < j \leq n} \frac{1}{x_i + x_j} \leq \frac{1}{4} \sum_{1 \leq i < j \leq n} \left(\frac{1}{x_i} + \frac{1}{x_j} \right).$$

Each term $1/x_i$ appears once with each of the other $n - 1$ indices, so this right-hand side equals the desired right-hand side.

This month's problem was solved by Brad Beadle ('96); Qi An; and Roger and Peter Bevan.

March 2018 Problem of the Month

Consider the sequences defined by $a_0 = 1$, $a_{n+1} = 2a_n + 1$, for integers $n \geq 0$, and b_0 real, $b_{n+1} = 2b_n - 1$, for integers $n \geq 0$. For what values of b_0 will $b_n > a_n$ hold for all nonnegative integers n ?

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

Editor: Rhonda Hatcher
Problem Editor: George Gilbert
Thought of the Month Editor: Robert Doran