TCU Math News Letter

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This view [of the infinite], which I consider to be the sole correct one, is held by only a few. While possibly I am the very first in history to take this position so explicitly, with all of its logical consequences. I know for sure that I shall not be the last!

- Georg Cantor

Editor: Dr. Rhonda Hatcher and Archive of Newsletters

Two Speakers in the TCU Lectureship Series in October

The next speaker in the TCU Mathematics Department Research Lectureship Series will be Professor Dave Witte of Oklahoma State University. He will present his talk, "Superrigidity: a Theorem in Group Theory," on Tuesday, October 1 at 4 p.m. This talk should be of particular interest to TCU undergraduates currently enrolled in Abstract Algebra. The second speaker this month will be Professor Nicolae Anghel of the University of North Texas. His talk, entitled "Index Theory for Fredholm Pairs," will be presented on Tuesday, October 22 at 4 p.m.

All of the TCU Lectureship talks are presented in Winton Scott Hall 145. Refreshments are served in the half-hour preceding each talk in Winton Scott Hall 112.

Soap Bubbles at the Next Meeting of Parabola

Professor Ken Richardson will be the speaker at the next meeting of Parabola, the TCU undergraduate student organization, on Wednesday, October 16. Dr. Richardson will be assisted by Sarah Field, a senior mathematics major at TCU. Dr. Richardson will talk about "Soap Bubbles." He plans to discuss some of the mathematics of soap bubbles, and as you might expect from Dr. Richardson, he will be blowing some bubbles himself.

The Parabola meeting will begin with refreshments at 3:00 p.m. in Winton Scott Hall 112, and Dr. Richardson's talk will begin at 3:30 in Winton Scott Hall 145. All TCU students, faculty, and other members of community are invited to attend.

October 7 Deadline for Signing Up For the Putnam Exam

The Fifty-Seventh Annual William Lowell Putnam Mathematical Competition will be held on Saturday, December 7, 1996. This annual competition consists of a written exam with twelve questions. The questions require varying amounts of mathematical background, and all require a bit of ingenuity to solve. The scores on the exam are typically quite low, and even answering a few questions is considered a very good performance. The competition is open to undergraduates enrolled in colleges and universities of the United

States and Canada, who have not yet received a college degree. Any college or university with at least three entrants, can also enter the team competition. Prizes are awarded to the top twenty-five finishers and to the departments of mathematics of the institutions with the five top ranking teams.

Copies of last year's exam are posted on the Problem Solving bulletin board down the hall from the Mathematics Department.

The exam will be given at TCU in two parts on December 7. The first part will be given at 9:00 a.m. to 12:00 noon, and the second part at 2:00 p.m. to 5:00 p.m. All of the participants will be treated to a free pizza lunch.

The deadline for signing up to take the Putnam Exam this year is October 7. All TCU mathematics majors and others with a good mathematics background are encouraged to give it a try. Students who want to sign up should contact Professor George Gilbert. He is currently on leave so he can best be contacted by phone at 923-8146 or you can just leave him a note with your name and phone number in the Mathematics Department office.

Solution to the September 1996 Problem of the Month

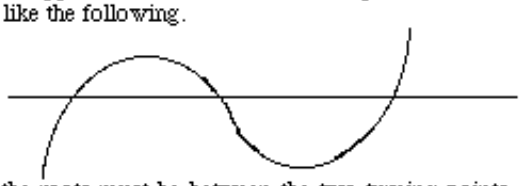
Problem: *Find all integers a for which the equation $x^3 - 7x + a = 0$ has three integer roots.*

Solution: Only $a = \pm 6$ lead to three integer roots. The following solution, edited to fit the column's space, is due to senior Brad Beadle. If $x^3 - 7x + a = 0$ has three integer roots, then for some integers b, c, and d,

$$x^3 - 7x + a = (x+b)(x+c)(x+d)$$
$$= x^3 + (b+c+d)x^2 + (bc+bd+cd)x + bcd\ .$$

Using $b + c + d = 0$ to eliminate d from $bc + bd + cd = -7$ and simplifying, we have $b^2 + bc + c^2 = 7$. Completing the square in c, yields $(c + b/2)^2 + 3b^2/4 = 7$, from which we find that $|b| \le \sqrt{28/3} = 3.06$. Because $-b$ is a root of $x^3 - 7x + a = 0$, we set -3, -2, -1, 0, 1, 2, and 3 in turn as roots, concluding that the only possibilities are $a = -6$, 0, 6. However, after factoring out the known root(s) in each case, we see that only -6 and 6 lead to three integer roots.

A slightly different approach uses calculus. If the given cubic has three real roots, its graph looks roughly like the following.



Observe that one of the roots must be between the two turning points. From the derivative $3x^2 - 7$ of $x^3 - 7x + a$, the turning points in our problem are $\pm\sqrt{7/3} = \pm 1.53$. Hence, one of $-1, 0$, and 1 must be a root of $x^3 - 7x + a = 0$. We continue as in Brad's solution.

Problem of the Month

A professor has two copies each of three versions of an exam. If the professor passes these exams out at random to a row of six students, what is the probability that some two adjacent students receive the same version of the exam?

Students and others are invited to submit solutions to Dr. George Gilbert (Math Dept. Office or P.O. 298900). Correct

solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

The TCU Math Newsletter will be published each month during the academic year. Dr. Hatcher: Editor; Dr. Gilbert: Problem Editor; Dr. Doran: Thought of the Month Editor. Items which you would like to have included should be sent to Dr. Hatcher (Math Dept. Office or P.O. 298900).