TCU Math News Letter

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What is that thing which does not give itself, and which if it were to give itself would not exist?

It is the infinite!

-- Leonardo da Vinci (1452-1519)

Editor: Dr. Rhonda Hatcher and Archive of Newsletters

TCU Lectureship Series Talks

On Tuesday, November 4, Professor Richard Stong of Rice University will present the talk "An Introduction to Four Manifold Topology." Another talk in the lectureship series will given by Professor Diane Hoffoss of Colorado College on Tuesday, November 11. Her talk is entitled "Using Foliations to Understand 3-Manifolds."

Both talks will begin at 4:00 p.m. in Winton Scott Hall 145. Refreshments will be served before the talks at 3:30 p.m. in Winton Scott Hall 171.

Next Parabola Meeting on Wednesday, November 19

Dr. Mostafa Ghandehari of the TCU Mathematics Department will speak about "Some Simple Arithmetic I Have Forgotten" at the next meeting of Parabola, the TCU undergraduate mathematics club. We will begin the meeting with refreshments at 3:00 p.m. in Winton Scott Hall 112 and the talk will begin at 3:30 p.m.

Opportunity for Undergraduates to Study in Budapest, Hungary

The *Budapest Semesters in Mathematics Program* enables junior and senior undergraduate students to spend a semester or year studying mathematics in Budapest, Hungary. All courses in the program are taught in English and the credits are transferable to North American colleges and universities. The instructors are members of Eötvös University and the Mathematical Institute of the Hungarian Academy of Sciences.

The program admits about 40 students per semester. The application deadline is April 30, 1998 for students wishing to attend next fall, and the deadline is November 1, 1998 for those wishing to attend in Spring 1999. Early applications (by as much as one year) are encouraged and will be processed promptly. For more information, look at the program brochure in the TCU Mathematics Department office or visit the Budapest Program web page Budapest Program web page.

Solution to the October 1997 Problem of the Month

Problem: Prove the following proposition: If a side of a triangle is less than the average of the other two sides, then the opposite angle is less than the average of the other two angles.

Solution: Ken Richardson and George Gilbert liked this one so much, they got carried away and came up with three proofs. Let a, b, and c denote the lengths of the sides of the triangle, and let A, B, and C denote the measure of the opposite angles (in degrees). The condition that C < (A+B)/2 is equivalent to C < (180 - C)/2, or C < 60.

I. From the law of cosines

$$\cos C = \frac{a^2+b^2-c^2}{2ab} > \frac{a^2+b^2-\left(\frac{a+b}{2}\right)^2}{2ab} = \frac{\frac{3}{4}a^2+\frac{3}{4}b^2-\frac{1}{2}ab}{2ab} = \frac{ab+\frac{3}{4}(a-b)^2}{2ab} \geq \frac{1}{2} = \cos 60,$$

or C < 60.

II. From the law of sines a = c sin A/ sin C and a = b sin A/ sin B. Also

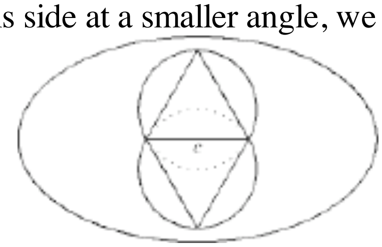
$$\sin A + \sin B = 2\sin((A+B)/2)\cos((A-B)/2) \leq 2\sin((A+B)/2)$$

for A and B between 0 and π. (This also follows because the graph of sin x is concave down between 0 and π.) Combining these observations with the hypothesis yields

$$\frac{1}{2} > \frac{C}{a+b} = \frac{\sin C}{\sin A + \sin B} \geq \frac{\sin C}{2\sin((A+B)/2)} = \frac{\sin C}{2\sin((180-C)/2)}$$
$$= \frac{\sin C}{2\sin(90-C/2)} = \frac{2\sin(C/2)\cos(C/2)}{2\cos(C/2)} = \sin(C/2)$$

or C/2 < 30, or C < 60.

III. Fixing the side of length c, the set of points forming a triangle with a given value of (a+b)/2 is an ellipse, with foci the two original vertices of the triangle. Since c < (a+b)/2, the two equilateral triangles with one side the given side lie inside the ellipse. Moreover the circles circumscribing these triangles lie inside the ellipse. Because the major arcs of these circles are all points subtending the fixed side at a 60 degree angle, and points outside the circle subtend this side at a smaller angle, we must have C < 60.



Problem of the Month

Show that there is a number with 1998 digits such that when the first (leading) digit is moved to the right of the rest of the numbers (i.e. to the ones places), the new 1998-digit number is exactly 3 times the first number.

Students and others are invited to submit solutions to Dr. George Gilbert (Math Dept. Office or P.O. 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will

be published in the next edition of the newsletter.

The TCU Math Newsletter will be published each month during the academic year. Dr. Hatcher: Editor; Dr. Gilbert: Problem Editor; Dr. Doran: Thought of the Month Editor. Items which you would like to have included should be sent to Dr. Hatcher (Math Dept. Office or P.O. 298900).