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TCU Math Newsletter

For every complex problem, there is a solution that is simple, neat, and wrong.

- H. L. Mencken

November Colloquium Talks

The Frank Stones Memorial Colloquium Lecture series will feature two talks in November 2011. The first talk, by Professor Ken Dykema of Texas A&M University, will be on Friday, November 4 at 3:30 pm. His talk is entitled "Sofic groups." In 1999, Gromov introduced an approximation property for infinite groups that essentially asks for finite subsets of the group to be approximated in a certain weak sense by sets in some finite permutation group. Groups satisfying this property are called sofic. At present, there are no known examples of non-sofic groups (though there are some intriguing candidates), and there are some results about classes of groups that are sofic. Professor Dykema will cover these topics and connections to some open problems about group rings and operator algebras

Professor Andy Putman of Rice University will present a second November talk in the lecture series on Friday, November 11 at 3:30 pm. In this talk, "The homology of congruence subgroups," Professor Putman will discuss the structure of the group homology of $SL_n(Z)$ and $\Gamma_n(p)$.

Both talks are in TUC 244 with refreshments served before the talk in TUC 300.

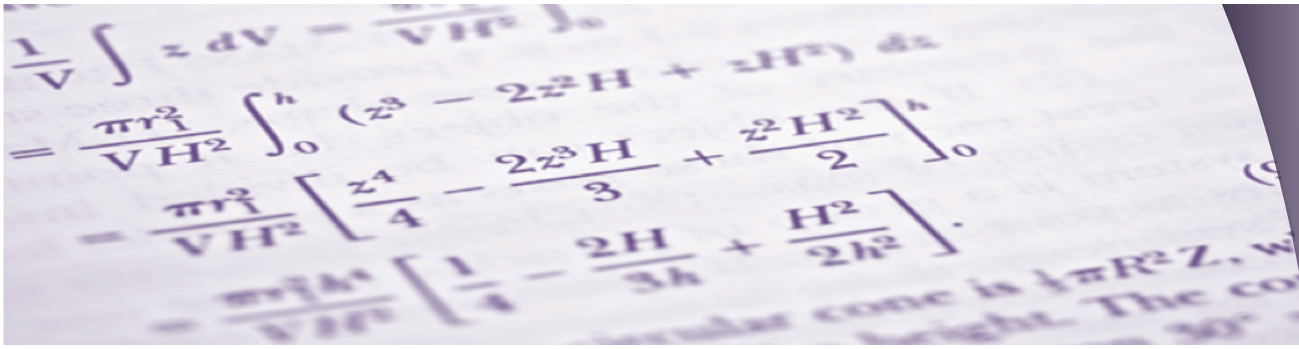
Professor Ken Richardson Recognized as Exemplifying the TCU Promise

At Fall Convocation, Chancellor Boschini focused on TCU's "culture of connection" and the TCU Promise. In the October 24 publication of *TCU This Week,* Professor Ken Richardson of Professor of Mathematics was recognized as exemplifying that promise

Dr. Richardson is a graduate of Rice University and has been a popular member of the TCU faculty since 1993. Ken regularly volunteers with Habitat for Humanity where he serves as a building site supervisor.

Using Mathematics to Detect Financial Bubbles

A financial bubble occurs when prices for something, such as housing or stocks, rise far above their actual value. In paper published last month in the *SIAM Journal on Financial Mathematics,* Robert Jarrow, Younes Kchia, and Philip Protter address the problem of detecting financial bubbles before they collapse. Their methods involve sophisticated volatility estimation techniques combined with the method of reproducing kernel Hilbert spaces. Their results confirm the suspicions of the presence of bubbles in many of the dot-com stocks of 1998–2001.



Solution to the October 2011 Problem of the Month

Problem: Let x, y, and z be real numbers such that $x + y + z = 1$ and $x^2 + y^2 + z^2 = 2$. What are the possible values of $x^3 + y^3 + z^3$?

Solution: The domain is the intersection of a sphere and a plane, which is a circle. Because the domain is a closed curve, the maximum and minimum values occur at critical points found by the method of Lagrange multipliers. We set

$$f(x,y,z) = x^3 + y^3 + z^3 - \alpha\left(x^2 + y^2 + z^2 - 2\right) - \beta(x + y + z - 1).$$

Taking derivatives, we see $3x^2 - 2\alpha x - \beta = 3y^2 - 2\alpha y - \beta = 3z^2 - 2\alpha z - \beta = 0$. Thus, x, y, and z are roots of the same quadratic polynomial, so some two, say y and z, must be equal. Substituting $x = 1 - 2y$ into $x^2 + 2y^2 = 2$, we find $6y^2 - 4y - 1 = 0$. Hence, $y = \dfrac{2 \pm \sqrt{10}}{6}$ and $x = \dfrac{1 \mp \sqrt{10}}{3}$. The minimum value of $x^3 + y^3 + z^3$ is $\dfrac{32 - 5\sqrt{10}}{18} = 0.8993\ldots$ and the maximum is $\dfrac{32 + 5\sqrt{10}}{18} = 2.6561\ldots$ (so $x^3 + y^3 + z^3$ cannot equal 3).

The October Problem of the Month as solved by Brad Beadle ('96).

November 2011 Problem of the Month

Is there a polynomial $p(x)$ of degree 2, with integer coefficients, whose value is irrational whenever x is an irrational number?

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

Editor: Rhonda Hatcher
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