TCU MATH NEWSLETTER

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To see a World in a Grain of Sand,
And a Heaven in a Wild Flower,
Hold Infinity in the palm of your hand
And Eternity in an hour.

- William Blake

Next Parabola Meeting on November 12

Duy Nguyen, a TCU mathematics major, will present the talk ***Energy of Graphs and Matrices*** at the next meeting of Parabola on Thursday, November 12. The talk will be held in Tucker Technology Center 244 at 4 pm, and refreshments will be served before the talk in TTC 300.

The energy of a graph originates in chemistry and was first defined by I. Gutman in 1978. The adjacent energy of a graph is defined by the sum of absolute values of eigenvalues of the adjacent matrix with respect to the graph. The concept of energy of a graph is important and has many applications in physics and chemistry. In his talk, Duy will present the results on Laplacian, signless Laplacian, and distance energy of a graph with a brief discussion of applications in chemistry and physics.

Duy's talk is the result of the work he did in the National Science Foundation Research for Undergraduates program in the summer of 2009 at Central Michigan University under the guidance of Professor Sivaram Narayan.

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Frank Stones Mathematics Research Lectureship

There will be two talks in November in the Frank Stones Mathematics Research Lectureship series.

The first talk will be presented by Professor Anna Spice of the TCU Mathematics Department. She will present the talk ***On the local (non-)extendibility of germs of holomorphic functions*** at 4 pm on Tuesday, November 3 in Tucker Technology Center 244.

Professor Prudence Heck of Rice University will present a talk at 4 pm on Tuesday, November 17. Check the TCU Mathematics Department web site at www.math.tcu.edu for more detailed information in the future.

All TCU students and faculty and other interested members of the community are invited to come to the talks. Refreshments will be served before each talk in TTC 300.

Problems and Solutions

Solution to the October 2009 Problem of the Month

Problem: Consider a sequence (a_n) satisfying $a_{n+1} = 2\, a_n^2 + 1$. What is the largest possible number of consecutive perfect squares in such a sequence?

The TCU Math Newsletter is published each month during the academic year.

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Solution: There cannot even be two consecutive perfect squares (unless one considers 0 a perfect square).

Consecutive squares would be a solution in positive integers to $y^2 = 2x^4 + 1$. Equivalently, $2x^4 = (y+1)(y-1)$. Since $y+1$ and $y-1$ are consecutive even positive integers, their greatest common factor is 2. Thus, one must be an even fourth power and the other twice a fourth power.

If $y + 1 = (2k)^4 = 16k^4$ and $y - 1 = 2m^4$, then $8k^4 = m^4 + 1$. However, m must be odd so that $m^4 + 1 = (m^4 - 1) + 2 = (m^2 + 1)(m + 1)(m - 1) + 2$ is 2 more than a multiple of 8, a contradiction.

Now suppose $y + 1 = 2k^4$ and $y - 1 = 16m^4$. Then $2(2m^2)^2 = 8m^4 = k^4 - 1 = (k^2 + 1)(k + 1)(k - 1)$. Noting that $k^2 + 1 = (k + 1)(k - 1) + 2$, we conclude all three factors are even with 2 as the greatest common divisor of any pair. Thus, each is either a perfect square or twice a perfect square. First observe that $k^2 + 1$ cannot be a perfect square so is twice a square. However, that makes $k + 1$ and $k - 1$ either both perfect squares or both twice a perfect square. In the first, case we have squares differing by 2 and, in the second, $(k + 1)/2$ and $(k - 1)/2$ are perfect squares whose difference is 1. Neither scenario is possible.

November 2009 Problem of the Month

Three distinct integers are chosen at random from $1, 2, ..., n$. What is the probability no two are consecutive?

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.