TCU MATH NEWSLETTER

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An expert is someone who knows some of the worst mistakes that can be made in his subject, and how to avoid them.

--- Werner Heisenberg

National Science Foundation Summer Research Opportunities for Undergraduates

Undergraduate mathematics majors who are interested in participating in one of the National Science Foundation Research Experiences in Mathematics for Undergraduates Sites in the summer of 2008 will need to begin the application process early in 2008. Some of the sites have application deadlines in February, and the applications typically require letters of recommendation from professors. The sites are located at universities across the country.

In these projects, undergraduate students conduct mathematical research under faculty guidance. They generally run from six to eight weeks in length. The student participants receive a stipend, housing support, and sometimes travel support. The experience gained in these REU projects is particularly helpful for students considering graduate study in mathematics.

Several TCU students have participated in REU programs in the past, including current undergraduate Darren Ong who was an REU participant both of the last two summers.

Undergraduates who are interested in learning more about or applying to one of the REU projects can find a list of NSF REU sites in mathematics with web links to the individual programs at www.nsf.gov/crssprgm/reu/reu_search.cfm.

Frank Stones Lectureship Talk on Tuesday, November 13

Professor Ralf Schmidt of the University of Oklahoma will be the next speaker in the Frank Stones Research Lectureship series at TCU. The talk, entitled ***Siegel modular new- and oldforms*** will be presented at 4:00 p.m. on Tuesday, November 13 in TTC 245. In the talk, Professor Schmidt will explain the concept of new- and oldforms in the classical theory of modular forms, and also in its local representation-theoretic version.

Refreshments will be served before the talk in TTC 300 at 3:30 p.m.

Upper Division Math Courses

For more information about some of the elective upper division math courses that are being offered in the Spring 2008 semester, see http://www.math.tcu.edu/newcourses.

The Math Newsletter Will Go Paperless

In the interest of the environment and more speedy delivery, beginning with the next edition, the primary format for the newsletter will be electronic. New issues of the TCU Math Newsletter will be announced via e-mail, and will be posted on the TCU Mathematics Department web site at www.math.tcu.edu/Newsletters/Archive.html.

If you are not a mathematics major at TCU or a faculty member of the TCU Mathematics Department, we may not have your e-mail address, so please send it to r.hatcher@tcu.edu so that you will be on our e-mail mailing list.

Problems and Solutions

Solution to the October 2007 Problem of the Month

Problem: Each of n balls is dropped into either box A, B, or C, with respective probabilities p, q, or r (so $p + q + r = 1$). Find the area of the region of probabilities (p, q, r) for which all n balls in box A is at least as likely as any of the other $(n^2 + 3n)/2$ possible distributions.

Solution: The probability of i balls in box A, j in box B, and k in box C is

$$P(i, j, k) = (n!/(i!j!k!))\, p^i q^j r^k .$$

In particular, we need $P(n, 0, 0) \geq P(n\text{-}1, 1, 0)$ and $P(n, 0, 0) \geq P(n\text{-}1, 0, 1)$. These two inequalities are equivalent to $p \geq nq$ and $p \geq nr$. But then

$$P(i,\ j,\ k) \leq n^{j+k\vee}\, p^i q^j r^k \leq p^n = P(n, 0, 0).$$

Our two inequalities, along with $p \geq 0$, $q \geq 0$, $r \geq 0$, and $p + q + r = 1$, define a quadrilateral with vertices (1, 0, 0), ($n/(n+1)$, $1/(n+1)$, 0), ($n/(n+1)$, 0, $1/(n+1)$), and ($n/(n+2)$, $1/(n+2)$, $1/(n+2)$). The diagonal at the vertex (1, 0, 0) divides the quadrilateral into two congruent triangles, each with two sides of lengths

$$((1\text{-}n/(n+1))^2+(1/(n+1))^2)^{½} = \sqrt{2}/(n+1)$$

and

$$((1\text{-}n/(n+2))^2 + (1/(n+2))^2 + (1/(n+2))^2)^{½} = \sqrt{6}/(n+2)$$

and with 30° included angle. Therefore, the area is

$$2\cdot½\cdot sin\ 30°\cdot \sqrt{2}/(n+1)\cdot \sqrt{6}/(n+2) = \sqrt{3}/((n+1)(n+2)) .$$

The October problem of the month was solved by undergraduate Darren Ong.

November 2007 Problem of the Month

Darren Ong continues to offer his challenge. If any TCU student (undergraduate or graduate) meets submits a correct solution to the Problem of the Month before he does, Darren will dye his "hair bubble-gum pink for at least a week."

Starting from $x = 0$, a fair coin is flipped n times. Each time a head occurs, 1 is added to x; after each tail, 1 is subtracted from x. Let a_n be the average (mean) of the absolute value of x after n flips. Does $a_n \rightarrow \infty$ as $n \rightarrow \infty$?

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.