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TCU Math Newsletter

One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers.
- Heinrich Hertz

Call for Abstracts for the College of Science and Engineering Student Research Symposium (SRS)

The TCU College of Science and Engineering Research Symposium (SRS) is a relaxed forum in which students can present their work in a poster presentation. SRS provides a nice opportunity to present work and also see what other students are doing. The posters are viewed by numerous faculty, students, and visitors. SRS will take place on Friday, April 20 in Tucker Technology Center.

Any TCU undergraduate or graduate student who has been engaged in some form of research is strongly encouraged to participate. The deadline for abstract submissions is Thursday, March 29.

For more information about SRS and to submit an abstract, visit the SRS website www.srs.tcu.edu.

Math Colloquium Talks

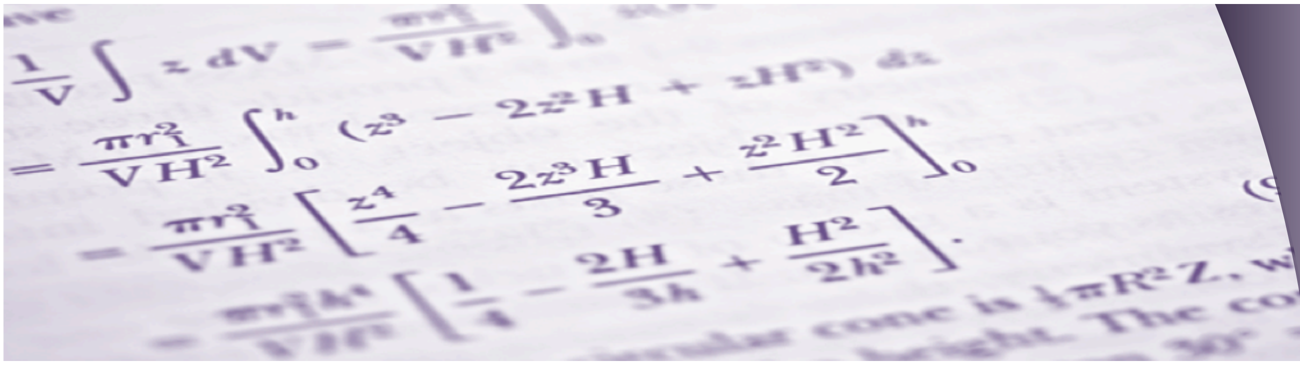
The Frank Stones Memorial Colloquium Lecture series will feature two speakers in March. Professor Tim Perutz of the University of Texas at Austin will present the talk “Arithmetic Geometry from Symplectic Topology” on Friday, March 2.
On Friday, March 30, Professor Brian Lehmann of Rice University of Texas will present “An Introduction to the Minimal Model Program.”

Both talks will take place at 3:30 pm in TUC 246 with refreshments at 3:00 pm in TUC 300. All students, faculty, and interested members of the community are welcome to attend.

Pi Day Celebration on March 14

Pi Day, a holiday commemorating the mathematical constant π, is celebrated nationally on March 14 because in month/date format 3/14 matches the first digits of π.

TCU’s Pi Day celebration will take place in the basement of Tucker from 11:30 am to 1:30 pm with free pizza for all. There will be a pizza eating contest from 12:30 to 1:00, and the winner will get to throw a pie in a professor’s face. There will also be a pie making contest that will be judged at 12:30 and a pinewood derby competition at 12 noon. There are no entry fees, and all students and faculty are invited to attend.



Solution to the February 2012 Problem of the Month

Problem: For $x \ge 0$, let $y = f(x)$ be continuously differentiable, with positive, increasing derivative. Consider the ratio between the distance from $(0, f(0))$ to $(x, f(x))$ along the curve $y = f(x)$ (the arc length from 0 to x) and the straight-line distance from $(0, f(0))$ to $(x, f(x))$. Must this ratio have a limit as $x \to \infty$?

Solution: This month's solution is a combination of that of Brad Beadle ('96) and Siggi Kurz and of the Editor. The limit is 1. Because replacing $f(x)$ with $f(x) - f(0)$ does not affect the conditions on the derivative, we may assume $f(0) = 0$.

The straight line distance is $D(x) = \sqrt{x^2 + [f(x)]^2}$. The arc length is $S(x) = \int_0^x \sqrt{1 + [f'(x)]^2}\, dx$. Note that $f'(x)$ is increasing so has a limit as $x \to \infty$. Suppose first that it has finite limit m. Because $f(x)/x$ is the average value of $f'(x)$ on $[0, x]$, it too goes to m. By l'Hôpital's rule,

$$\lim_{x\to\infty} \frac{S(x)}{D(x)} = \lim_{x\to\infty} \frac{\sqrt{1+[f'(x)]^2}\sqrt{x^2+[f(x)]^2}}{x + f'(x)f(x)} = \lim_{x\to\infty} \frac{\sqrt{1+[f'(x)]^2}\sqrt{1+[f(x)/x]^2}}{1 + f'(x)f(x)/x} = 1.$$

Now suppose $f'(x)$ and $f(x)/x$ go to ∞. The bounds

$$1 \le \frac{S(x)}{D(x)} \le \frac{\int_0^x (1 + f'(x))\, dx}{\sqrt{x^2 + [f(x)]^2}} = \frac{(x + f(x))}{\sqrt{x^2 + [f(x)]^2}} = \frac{(1 + f(x)/x)}{\sqrt{1 + [f(x)/x]^2}}.$$

imply $S(x)/D(x) \to 1$ by squeezing.

Brian Preskitt submitted a similar solution.

March 2012 Problem of the Month

Show that there is a positive integer n with first (leading) digit 1, such that n^2 has first digit 2, n^3 has first digit 3, and n^4 has first digit 4. Show that is not possible to further have 5 as the first digit of n^5.

Students and others are invited to submit solutions to Dr. George Gilbert by e-mail (g.gilbert@tcu.edu) or hard copy (Math Dept. Office or TCU Box 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.