TCU Math News Letter

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We live on a placid island of ignorance in the midst of black seas of infinity, and it is not meant that we should voyage far.

--- H. P. Lovecraft

Editor: Dr. Rhonda Hatcher and Archive of Newsletters

Lyndsey Blott Named Senior Scholar of the Mathematics Department

The 2001 Senior Scholar of the Mathematics Department will be Lyndsey Blott. Lyndsey will receive this honor at the Honors Banquet on Thursday, April 5. Congratulations Lyndsey!

Next Parabola Meeting on Wednesday, April 11

Pizza and a movie about chaos and dynamics will be the featured attractions at the next meeting of Parabola, the TCU undergraduate mathematics club. The meeting will be held on Wednesday, April 11, 4:30-5:30 p.m. in Winton Scott Hall 145. All TCU students, faculty, and friends of the Mathematics Department are invited to attend.

Calculus Bee on April 19

There is still time left to sign up for the TCU Mathematics Department Calculus Bee, which will be held on Thursday, April 19 beginning at 4:00 p.m. in Winton Scott Hall 145. Refreshments will be served in Winton Scott Hall 171 at 3:30 p.m. Students interested in competing in the Calculus Bee should sign up in the Mathematics Department Office in Winton Scott Hall 112.

All TCU undergraduates are eligible to compete. A cash prize of $75 will be awarded to the first place finisher, $50 will go to the second place finisher, and the third place finisher will be awarded $25.

Solution to the March 2001 Problem of the Month

Suppose that we wish to know which windows in a 36-floor building are safe to drop eggs from, and which will cause the eggs to break on landing. We assume: - The effect of a fall is the same for all eggs. An egg that survives a fall can be used again. A broken egg must be discarded. -If an egg breaks when dropped, then it would break if dropped from a higher window. If an egg survives a fall then it would survive a shorter fall. (It is not ruled out that the first-floor windows break eggs, nor is it ruled out that the 36th-floor

windows do not cause an egg to break.) If only one egg is available and we wish to be sure of obtaining the right result, the experiment can be carried out in only one way. Drop the egg from the first-floor window; if it survives, drop it from the second floor window. Continue upward until it breaks. In the worst case, this method may require 36 droppings. Suppose 2 eggs are available. What is the least number of egg-droppings that is guaranteed to work in all cases? (A Macalester College Problem of the Week.)

Solution: The least number of droppings needed in the worst case scenario is eight. The key observation is that once the first egg breaks, the second egg must be dropped from the lowest level at which it could possibly break given the evidence from prior droppings. We will use mathematical induction to show that at most n droppings are required for a building with between $(n-1)n/2+1$ and $n(n+1)/2$ floors ($n(n+1)/2$ is the "nth triangular number") and that sometimes n droppings are needed. The case $n=1$ applies only to one-floor buildings, for which it is clear that exactly one dropping is always required. Assume the claim for a worst case of $1, 2, \ldots, n$ droppings. We will verify the claim for a worst case of $n+1$ droppings. To see that $n+1$ droppings are sufficient for between $n(n+1)/2+1$ and $(n+1)(n+2)/2$ floors, suppose we drop the first egg from the $(n+1)$st floor. If it breaks, we drop the second egg from the first floor and move up, possibly taking as many as n+1 droppings in all. If not, floors $n+2$ and up essentially form a building with between $n(n+1)/2+1-(n+1)=(n-1)n/2+1$ and $(n+1)(n+2)/2-(n+1)=n(n+1)/2$ floors, allowing us to apply the inductive hypothesis. If we started any higher and the egg breaks, we could require more than n additional droppings. If we started lower and the egg fails to break, we could require at least n more droppings.

Math major Dustin Sitar found this optimal strategy.

Problem of the Month

This month's problem originally appeared in the Journal of Recreational Mathematics and is due to Sidney Kravitz. It seems particularly appropriate for this month.

In Erewhon, income is taxed by both the state and federal governments. Each government recognizes the other's tax as the only deduction from taxable income. Furthermore, the tax must be paid during the year in which it accrues. If the federal tax rate is 20% and the state rate is 10%, how much tax should be paid to each on an income of 10,000?

(Remember that math majors will earn 10 points in the Bucks for Books lottery for a correct solution. For details and other ways to earn points, refer to the September 2000 Newsletter or visit the web page www.math.tcu.edu/math/BucksForBooks.html.)

Students and others are invited to submit solutions to Dr. George Gilbert (Math Dept. Office or P.O. 298900). Correct solutions submitted by persons who are not members of the TCU math faculty will be acknowledged in the next issue of the newsletter. Note that a correct solution is an answer and a justification of its correctness. The solution to the problem will be published in the next edition of the newsletter.

The TCU Math Newsletter will be published each month during the academic year. Dr. Hatcher: Editor; Dr. Gilbert: Problem Editor; Dr. Doran: Thought of the Month Editor. Items which you would like to have included should be sent to Dr. Hatcher (Math Dept. Office or P.O. 298900).